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# Absent or overlooked? <br> Approaches to overcome the problem of non-detection in forest inventories 

## Ausente ou negligenciado?

Procedimentos para superar a não detecção nos inventários florestais

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## Non-detection

- The emphasis of traditional forest inventories is on timber production
- However, with increasing interest, society looks the forests as future carbon sinks, potential biomass energy resources, wildlife habitat, water resources, and for other ecosystem services, so that the demand for assessing these aspects has increased (Ducey, 2014; Kenning et al., 2005)
- Traditional sampling techniques for forest inventories use a census within a limited search area for inference
- When sampling rare objects, these sampling techniques may proof to be inefficient and cost intensive due to their limited search area
- Total detectability of objects is assumed, any violation of this assumption leads to a non-detection bias
- The problem of non-detection becomes especially pronounced when sampling
- rare objects (e.g. rare and valuable tree species, or rare and ecologically important objects like snags)
- in highly structured forests
- with limited sighting conditions


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## Distance sampling - an alternative method to overcome 气 the problem of non-detection

- Widely used for estimating the abundance of all kinds of biological populations, especially birds and mammals Thomas et al. (2012)
- Based on the work of Anderson \& Pospahala (1970); further developed over the years; standard text books by Buckland et al. $(2001,2004)$
- Two main methods
- Line transect sampling (LTS)
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- Information used for inference is not a census within a limited search area (e.g. within a circular sample plot), but the recorded distances to detected objects of interest obtained by surveying lines or points respectively (Marques et al., 2011)
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## Distance sampling applications in forestry...

...are -still- quite rare:

Point transect sampling (PTS)

- Deadwood volume and carbon storage (Ritter \& Saborowski, 2010, 2012, 2014)
- Bias correction of angle count sampling (Ritter et al., 2013)
- Bias correction of terrestrial laser scanning (Ducey \& Astrup, 2013; Astrup et al., 2014)

Line transect sampling (LTS)

- Habitat trees (Bäuerle et al., 2009; Didas, 2009; Bäuerle \& Nothdurft, 2011)
- Low abundance tropical tree species (Kissa \& Sheil, 2012)
- Logging damage Siebert \& Ritter (in preparation)

Introduction
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Ritter, T.; Saborowski, J. (2012): Point transect sampling of deadwood: a comparison with well-established sampling techniques for the estimation of volume and carbon storage in managed forests. In: European Journal of Forest Research 131(6): 1845-1856, doi:
10.1007/s10342-012-0637-2

Ritter, T.; SABOROWSKI, J. (2014): Efficient integration of a deadwood inventory into an existing forest inventory carried out as 2-phase sampling for stratification. In: Forestry 87(4): 571-581, doi: 10.1093/forestry/cpu016

## Lower Saxony state forest district inventory (BI)

Two-phase sampling for stratification (2-SS) in a cycle of 10 years (Böckmann et al., 1998).

Phase I:

- Systematic sampling grid ( $100 \mathrm{~m} \times 100 \mathrm{~m}$ )
- Interpretation of CIR arial images

Phase II:

- Random selection of sample points within each stratum
- Two concentric circular sample plots (6 m and 13 m radius)

| Dominating | Age class |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| species group | $0-40$ | $>40-80$ | $>80-120$ | $>120$ |
| Decidous | dec1 | dec2 | dec3 | dec4 |
| Coniferous | con1 | con2 | con3 | con4 |

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## Area under investigation

2416 ha, located in the heart of Germany:


State forest district
Reinhausen

- Sub district Reinhausen
- Sub district Sattenhausen
Subsample of the BI
- First inventory in summer (235 plots)
- Repeated inventory in winter (228 plots)


## Pilot study

## Sampling techniques

Downed coarse woody debris
$\left(d_{\max } \geq 7 \mathrm{~cm}\right)$

- Fixed area sampling (FAS) on 13 m radius plots
- Line Intersect Sampling (LIS)
- Point Transect Sampling (PTS)

Standing deadwood
(DBH $\geq 7 \mathrm{~cm}$ )

- Fixed area sampling (FAS) on 13 m radius plots
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## Measurements

- Tree species (if possible)
- Decay class (using the key of Müller-Using \& Bartsch, 2009)
- DBH
- Height


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## Point Transect Sampling

## Detection function



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## Detection probability

$$
\begin{align*}
\hat{P}_{a_{h}} & =\frac{\int_{0}^{\omega} g(r) 2 \pi r d r}{\pi \omega^{2}} \\
& =\frac{2}{\omega^{2}} \int_{0}^{\omega} r \cdot \hat{g}(r) d r \tag{1}
\end{align*}
$$

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Object density

$$
\begin{equation*}
\hat{D}_{h}=\frac{m_{h}}{n_{h} \pi \omega^{2} \hat{P}_{a_{h}}} \tag{2}
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Volume

$$
\begin{equation*}
\hat{\bar{Y}}_{h}=\hat{D}_{h} \cdot \hat{E}(s) \tag{3}
\end{equation*}
$$

## Detection probability within the different strata



## Volume estimation

三

| Sampling <br> campaign | Sampling <br> technique | $\hat{Y}$ <br> $\left[\mathrm{~m}^{3} \mathrm{ha}^{-1}\right]$ | $\mathrm{SE}(\hat{Y})$ <br> $\left[\mathrm{m}^{3} \mathrm{ha}^{-1}\right]$ | $\hat{T}$ <br> $[\mathrm{~min}]$ |
| :---: | :---: | ---: | ---: | ---: |
| Summer $(\mathrm{n}=235)$ | FAS | 2.54 | 0.56 | 421 |
|  | PTS | 3.04 | 0.39 | 831 |
| Winter $(\mathrm{n}=228)$ | FAS | 3.86 | 1.15 | 413 |
|  | PTS | 3.05 | 0.42 | 923 |

## Optimization

| Sampling <br> campaign | Sampling technique | $n$ | SE $(\hat{Y})$ <br> $\left[\mathrm{m}^{3} \mathrm{ha}^{-1}\right]$ | $\hat{T}$ <br> $[\mathrm{~min}]$ |
| :--- | :--- | ---: | ---: | ---: |
| Summer | FAS (All ph-2 plots) | 600 | $\mathbf{0 . 3 6 6}$ | $\mathbf{1 0 6 2}$ |
|  | PTS (All ph-2 plots) | 600 | 0.175 | 2148 |
|  | PTS (Optimal allocation) | 79 | 0.365 | 301 |
| Winter | FAS (All Ph-2 plots) | 600 | 0.599 | 1085 |
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## Originally published as:

Ritter, T.; Nothdurft, A.; Saborowski, J. (2013): Correcting the nondetection bias of angle count sampling. In: Canadian Journal of Forest Research 43(4): 344-354, doi: 10.1139/cjfr-2012-0408

## Approach 1

## Heuristic

- Expand each tree count by the tree's individual inverse estimated detection probability to correct for the negative bias introduced by overlooking trees.


## Additional sampling effort

- The distance $r_{j}$ from the plot center to each sighted tree, which is supposed to be counted by ACS, has to be measured.


## Estimator

$$
\begin{equation*}
\hat{G}_{B C A C S 1}=k \cdot \sum_{j=1}^{z_{i}} \frac{1}{\hat{g}\left(r_{j}\right)} \tag{5}
\end{equation*}
$$

## Approach 2

## Heuristic

- Expand each tree count by the inverse estimated mean detection probability of all trees which have the same DBH $d_{j}$ (and therefore also the same marginal inclusion circle $K_{j}$ ) and are supposed to be counted at any sample point.

Additional sampling effort

- The diameter of each counted tree has to be measured.
- Measuring all distances $r_{j}$ is not necessary, as long as enough measurements are taken to estimate $g(r)$.
Mean detection probability within the marginal inclusion circle
- The radius of the marginal inclusion circle is $R_{j}=d_{j} /(2 \sqrt{k})$.
- The probability to detect a tree with DBH $d_{j}$ from a random point within its marginal inclusion circle $K_{j}$ can be estimated by $\hat{P}_{a_{j}}=\frac{2}{R_{j}^{2}} \int_{0}^{R_{j}} r g(r) d r$


## Estimator

$$
\begin{equation*}
\hat{\bar{G}}_{B C A C S 2}=k \cdot \sum_{j=1}^{z} \frac{1}{\hat{P}_{a_{j}}}=\sum_{j=1}^{z} \frac{d_{j}^{2}}{4 R_{j}^{2} \hat{P}_{a_{j}}} \tag{6}
\end{equation*}
$$

## Theoretical justification

The Horvitz-Thompson estimator of the total of $Y$ over $N$ trees, is given by

$$
\begin{equation*}
\hat{Y}(x)=\sum_{j=1}^{z} \frac{Y_{j}}{\pi_{j}} \text { with } \pi_{j}=\frac{\pi R_{j}^{2}}{A^{*}} \tag{7}
\end{equation*}
$$

$A^{*}=$ Inventory area extended by the peripheral zone (Mandallaz, 2008)

$$
R_{j}=d_{j} /(2 \sqrt{k})
$$

As trees may be overlooked, the inclusion probability $\pi_{j}$ must be corrected:

$$
\begin{align*}
\boldsymbol{\pi}_{j}^{+} & =P\left(\left\{x \in K_{j}\right\} \cap\{\mathrm{j} \text { is detected }\}\right) \\
& =P\left(x \in K_{j}\right) P\left(\mathrm{j} \text { is detected } \mid x \in K_{j}\right)=\boldsymbol{\pi}_{j} P_{a_{j}} \tag{8}
\end{align*}
$$

This leads to the unbiased estimator

$$
\begin{equation*}
\hat{Y}(x)=\frac{1}{A^{*}} \sum_{j=1}^{z} \frac{Y_{j}}{\pi_{j}^{+}}=k \sum_{j=1}^{z} \frac{Y_{j}}{(\pi / 4) d_{j}^{2} P_{a_{j}}} \tag{9}
\end{equation*}
$$

of the $Y$ total per area unit.

## Application to basal area density estimates

## BcACS2

- If the response variable $Y$ is the basal area density $\bar{G}$, the corrected Horvitz-Thompson estimator can be simplified to

$$
\begin{equation*}
\hat{\bar{G}}(x)=k \sum_{j=1}^{z} \frac{1}{P_{a_{j}}} \tag{10}
\end{equation*}
$$

Replacing $P_{a_{j}}$ by $\hat{P}_{a_{j}}$ leads to the approx. unbiased estimator $\hat{\bar{G}}_{B C A C S 2}$

## BcACS1

- Under the assumption of CSR, and if $x \in K_{j}$ for a tree with DBH $d_{j}$, it holds

$$
\begin{equation*}
E\left(g\left(r_{j}\right) \mid d_{j}\right)=\frac{1}{\pi R_{j}^{2}} \int_{0}^{R_{j}} g(r) 2 \pi r d r=P_{a_{j}} \tag{11}
\end{equation*}
$$

Thus, $\hat{\bar{G}}_{\text {BcACS1 }}$ can also approx. correct for the nondetection bias in ACS.

## Performance of the estimators






Simulation study

- Poisson distributed trees
- Simple random sampling
- $k=1$ for all ACS-estimators
- "Density" represents the Gaussian kernel density estimation of the probability distribution function of $\hat{\bar{G}}$.Application of point transect sampling in forest inventories



## Correcting the non-detection bias of angle count sampling (ACS)

4 Correcting the non-detection bias of terrestrial laser scanning (TLS) in a single-scan-modeConclusions

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## Sample sites \& data acquisition

Sample sites

- 12 mature forest stands in southern Norway
- dominated by Norway spruce (Picea abies L.) and Scots pine (Pinus sylvestris L.)
- varying mixtures of broadleaved species (mainly birch (Betula pubescens Ehrh. and Betula pendula Roth.))
Data acquisition
- $20 \mathrm{~m} \times 20 \mathrm{~m}$ inventory grid
- FAS on $250 \mathrm{~m}^{2}$ sample plots
- FARO LS 880
- Tripod-mounted
- Full horizontal scan ( $360^{\circ}$ horizontal and $320^{\circ}$ vertical fields of view)
- Resolution of $0.009^{\circ}$ (vertical) and $0.00076^{\circ}$ (horizontal).
- Truncation points at 8.92 m and 15 m


## Data analysis

- Stem extraction was done by a commercial TLS operator (Treemetrics Ltd.) using their proprietary software Autostem Forest
- A list of all detected trees, including their position and the diameter of the stem estimated for each 10 cm section was provided for all plots
- This data set can be treated as a FAS sample (uncorrected TLS)
- As the scanner-tree distances of all detected trees are known, the dataset can also be treated as a PTS sample (bias corrected TLS)


## Comparison of stand-level volume estimates



Comparison of the FAS reference values with:

- Uncorrected TLS data (A \& B)
- Bias corrected TLS data (C \& D)

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## Conclusions

- PTS (and distance sampling in general) is quit new to forestry, but is well established in other fields
- PTS is a very efficient and cost-saving sampling technique for rare objects
- PTS can easily be integrated into existing forest inventories
- PTS-theory can be annlied to existing inventory technioues (ACS \& TLS) to correct for non-detection

> In my opinion, PTS (and distance sampling in general) is worth to be tested in other forestry related applications maybe, you have some ideas, I would be very happy to establish an extensive cooperation!

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- PTS is a very efficient and cost-saving sampling technique for rare objects
- PTS can easily be integrated into existing forest inventories
- PTS-theory can be applied to existing inventory techniques (ACS \& TLS) to correct for non-detection

> In my opinion, PTS (and distance sampling in general) is worth to be tested in other forestry related applications maybe, you have some ideas, I would be very happy to establish an extensive cooperation!

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## Obrigado pela sua atenção!

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## Liegendes Totholz - Vorrat

$\underline{\Delta}$

| Type <br> of DW | Sampling <br> campaign | Sampling <br> technique | $\hat{Y}$ <br> $\left[\mathrm{~m}^{3} \mathrm{ha}^{-1}\right]$ | $\mathrm{SE}(\hat{\bar{Y}})$ <br> $\left[\mathrm{m}^{3} \mathrm{ha}^{-1}\right]$ |
| :--- | :--- | :--- | ---: | ---: |
| CWD $(d \geq 15 \mathrm{~cm})$ | Summer | FAS | 8.54 | 0.95 |
|  |  | LIS | 7.73 | 1.18 |
|  | Winter | FAS | 7.97 | 0.99 |
| CWD $(d \geq 7 \mathrm{~cm})$ |  | LIS | 7.96 | 1.18 |
|  |  | FAS | 13.10 | 1.10 |
|  |  | LIS | 13.90 | 1.37 |

## Liegendes Totholz - Effizienzvergleich

| Type of DW | Sampling campaign | Sampling technique | $N$ | $\begin{array}{r} \hat{T} \\ {[\mathrm{~min}]} \end{array}$ | $\begin{array}{r} \mathrm{SE}(\hat{Y}) \\ {\left[\mathrm{m}^{3} \mathrm{ha}^{-1}\right]} \end{array}$ | $\mathrm{CV}(\hat{Y})$ [\%] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| CWD$(d \geq 15 \mathrm{~cm})$ | Summer | LIS (all ph-2 points) | 600 | 3626 | 0.682 | 8.8 |
|  |  | FAS (all ph-2 points) | 600 | 8471 | 0.535 | 6.3 |
|  |  | FAS (optimal allocation) | 285 | 4086 | 0.682 | 8.0 |
|  |  | FAS (allocation prop. to ph-1) | 311 | 4398 | 0.682 | 8.0 |
|  |  | FAS (allocation prop. to ph-2) | 367 | 5176 | 0.682 | 8.0 |
|  | Winter | LIS (all ph-2 points) | 600 | 3053 | 0.681 | 8.6 |
|  |  | FAS (all ph-2 points) | 600 | 5866 | 0.549 | 6.9 |
|  |  | FAS (optimal allocation) | 323 | 3176 | 0.681 | 8.6 |
|  |  | FAS (allocation prop. to ph-1) | 356 | 3420 | 0.681 | 8.6 |
|  |  | FAS (allocation prop. to ph-2) | 387 | 3788 | 0.681 | 8.6 |
| $\begin{aligned} & \text { CWD } \\ & (d \geq 7 \mathrm{~cm}) \end{aligned}$ | Summer | LIS (all ph-2 points) | 600 | 3918 | 0.793 | 5.7 |
|  |  | FAS (all ph-2 points) | 600 | 15747 | 0.622 | 4.8 |
|  |  | FAS (optimal allocation) | 277 | 7566 | 0.793 | 6.1 |
|  |  | FAS (allocation prop. to ph-1) | 302 | 8213 | 0.793 | 6.1 |
|  |  | FAS (allocation prop. to ph-2) | 368 | 9656 | 0.793 | 6.1 |

## "'Zero-Inflation"'



## Auswahl der Entdeckungsfunktion

AIC basierte Modellauswahl

- $\hat{g}(r) \propto \operatorname{key}(r)[\operatorname{series}(r)]$
- Straten als Kovariate
- Alle möglichen Kombinationen von Schlüsselfunktion (key) und seriellem Anpassungsterm max. 5 Grades (series)


## Schlüsselfunktionen

- Uniform: $\hat{g}(r)=1 / \omega$
- Halb-Normal: $\hat{g}(r)=e^{-0,5 r^{2} / \sigma^{2}}$
- Hazard-Rate:

$$
\hat{g}(r)=1-e^{-(r / \sigma)^{-b}}
$$

## Serielle Anpassungsterme

- Cosin: $\sum_{k=2}^{q} a_{k} \cos \left(\frac{k \pi r}{\omega}\right)$
- Polynominal: $\sum_{k=2}^{q} a_{k}\left(\frac{r}{\omega}\right)^{2 k}$
- Kein Anpassungsterm


## Wahrscheinlichkeitsdichte

Anpassung von $\hat{f}(r)$ an die empirischen Daten,

$g(r)$ ergibt sich dann aus

$$
g(r)=\frac{r \cdot f^{\prime}(0)}{f(r)}
$$

Die Objektdichte kann direkt aus der pdf geschätzt werden:

$$
\hat{D}=\frac{m \cdot \hat{f}^{\prime}(0)}{2 \pi n}
$$

da

$$
P_{a}=\frac{2}{\omega^{2} f^{\prime}(0)}
$$

## Schätzung des mittleres Objektvolumen

三

## Log-Transformation:

$$
z_{i}=\log _{e}\left(s_{i}\right)
$$

Mittleres (transformiertes) Volumen der entdeckten Objekte:

$$
\hat{E}_{d}(z \mid r)=a+b \cdot \hat{g}(r)
$$

Mittleres Objektvolumen:

$$
\begin{gathered}
\hat{E}(z)=\hat{E}_{d}(z \mid r=0)=a+b \\
\hat{E}(s)=e^{a+b+\widehat{\operatorname{var}}(\hat{z}) / 2}
\end{gathered}
$$

## Analytische Varianzschätzung beim PTS

Die Varianzschätzung beim PTS erfolgt nach der Delta-Methode (Seber, 1982; zitiert nach Buckland et al., 2001):

$$
\widehat{\operatorname{var}}\left(\hat{\bar{Y}}_{h}\right)=\hat{\bar{Y}}_{h}^{2} \cdot\left(\frac{\widehat{\operatorname{var}}\left(m_{h}\right)}{m_{h}^{2}}+\frac{\widehat{\operatorname{var}}\left(a \cdot \hat{P}_{a_{h}}\right)}{\left(a \cdot \hat{P}_{a_{h}}\right)^{2}}+\frac{\widehat{\operatorname{var}}\left(\hat{E}_{h}(s)\right)}{\left(\hat{E}_{h}(s)\right)^{2}}\right)
$$

mit

$$
\begin{array}{ll}
\widehat{\operatorname{var}}\left(m_{h}\right) & =\frac{1}{n_{h}\left(n_{h}-1\right)} \sum_{i=1}^{n_{h}}\left(m_{h i}-\bar{m}_{h}\right)^{2} \\
\widehat{\operatorname{var}}\left(a \cdot \hat{P}_{a_{h}}\right) & =\frac{1}{m_{h} \hat{\sigma}_{h}^{4}} \\
\widehat{\operatorname{var}}\left(\hat{E}_{h}(s)\right) & =e^{2(a+b)+\widehat{\operatorname{var}(\hat{z})} \cdot\left(1+\frac{\widehat{\operatorname{var}}(\hat{z})}{2}\right) \cdot \frac{\widehat{\operatorname{var}}(\hat{z})}{m_{h}}}
\end{array}
$$

## Bootstrap Varianzschätzung beim PTS

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Die Bootstrapvarianz (Davison et al., 1986) kann analytisch aus den Bootstrapvarianzen der einzelnen Komponenten zusammengesetzt werden:

$$
\widehat{\operatorname{var}}_{B 1}\left(\hat{Y}_{h B}\right)=\hat{Y}_{h B}^{2} \cdot\left(\frac{\widehat{\operatorname{var}}_{h B}\left(m_{h B}\right)}{m_{h B}^{2}}+\frac{\widehat{\operatorname{var}}_{B}\left(a \cdot \hat{P}_{a_{h} B}\right)}{\left(a \cdot \hat{P}_{a_{h} B}\right)^{2}}+\frac{\widehat{\operatorname{var}}_{B}\left(\hat{E}(s)_{B}\right)}{\left(\hat{E}(s)_{B}\right)^{2}}\right)
$$

Alternativ kann sie direkt geschätzt werden (Buckland et al., 2001):

$$
\widehat{\operatorname{var}}_{B 2}\left(\hat{\bar{Y}}_{h B}\right)=\frac{\sum_{i=1}^{B}\left(\hat{\bar{Y}}_{h(i)}-\hat{\bar{Y}}_{h B}\right)^{2}}{B-1}
$$

## Konfidenzintervalle

Analytisches Konfidenzintervall für $\bar{Y}_{h}$ :

$$
C I_{A(\bar{Y})}=\left[\frac{\hat{Y}_{h}}{e^{z_{\alpha} \cdot \sqrt{\widehat{\operatorname{var}}\left(\log _{e} \hat{Y}_{h}\right)}}} ; \quad \hat{Y}_{h} \cdot e^{z_{\alpha} \cdot \sqrt{\widehat{\operatorname{var}\left(\log _{e} \hat{Y}_{h}\right)}}}\right]
$$

unter der Voraussetzung $m_{i} \stackrel{\text { i.i.d. }}{\sim} \operatorname{LN}\left(\mu, \sigma^{2}\right)$ (Burnham et al., 1987).
Bootstrap-Konfidenzintervall für $\bar{Y}_{h}$ :

$$
C I_{B(\bar{Y})}=\left[\begin{array}{ll}
\hat{Y}_{h(B+1) \alpha} & ; \hat{\bar{Y}}_{h(B+1)(1-\alpha)}
\end{array}\right]
$$

unter der Voraussetzung $m_{i} \stackrel{\text { i.i.d. }}{\sim} \mathrm{D}$

## Softwarebug bei der Schätzung von var(m)

Designbasierter Varianzschätzer P2 (Fewster et al., 2009):

$$
\widehat{\operatorname{var}}_{P 2}\left(\frac{1}{n} \sum_{r=1}^{n} \frac{m_{r}}{t_{r}}\right)=\frac{1}{n(n-1)} \sum_{i=1}^{n}\left(\frac{m_{i}}{t_{i}}-\frac{1}{n} \sum_{r=1}^{n} \frac{m_{r}}{t_{r}}\right)^{2}
$$

$P 2$ gewichtet Stichprobenpunkte unabhängig von ihrem $t_{i}$ gleich. Modellbasierter Varianzschätzer P3 (Fewster et al., 2009):

$$
\widehat{\operatorname{var}}_{P 3}\left(\frac{m}{T}\right)=\frac{1}{T(m-1)} \sum_{i=1}^{m} t_{i}\left(\frac{m_{i}}{t_{i}}-\frac{m}{T}\right)^{2}
$$

$P 3$ gewichtet Stichprobenpunkte mit hohem $t_{i}$ stärker als solche mit niedrigem $t_{i}$.

## Softwarebug bei der Schätzung von var(m)

Gleichheit der Schätzer Wenn $t_{i}=t$ (für alle $i$ ) gilt (Fewster et al., 2009):

$$
\widehat{\operatorname{var}}_{P 1}\left(\frac{m}{n t}\right)=\widehat{\operatorname{var}}_{P 2}\left(\frac{1}{n} \sum_{r=1}^{n} \frac{m_{r}}{t_{r}}\right)=\widehat{\operatorname{var}} P 3\left(\frac{m}{T}\right)=\widehat{\operatorname{var}}\left(\frac{\bar{m}}{t}\right)=\frac{1}{t^{2} k(k-1)} \sum_{i=1}^{k}\left(n_{i}-\bar{n}\right)^{2}
$$

Wenn $t_{i}=1$ (für alle $i$ ) gilt, ist $T=n$ und somit eine weitere Vereinfachung möglich:

$$
\widehat{\operatorname{var}}_{P 1}\left(\frac{m}{n t}\right)=\widehat{\operatorname{var}}\left(\frac{m}{n}\right)=\frac{1}{n(n-1)} \sum_{i=1}^{n}\left(m_{i}-\bar{m}\right)^{2}
$$

Im Falle der von uns durchgeführten Totholzinventur ist (innerhalb einer Aufnahmekampagne) $t_{i}=1$ (für alle $i$ ) und somit

$$
\widehat{\operatorname{var}}_{P 1}\left(\frac{m}{n t}\right)=\widehat{\operatorname{var}}_{P 2}\left(\frac{1}{n} \sum_{r=1}^{n} \frac{m_{r}}{t_{r}}\right)=\widehat{\operatorname{var}}_{P 3}\left(\frac{m}{T}\right)=\widehat{\operatorname{var}}\left(\frac{m}{n}\right)=\frac{1}{n^{2}} \widehat{\operatorname{var}}(m)
$$

## Arbeitszeiten

## Grundidee:

Einteilung in

- Entscheidungsrelevante Arbeitszeit (Suchen und Vermessen der Totholzobjekte am Stichprobenpunkt)
- Entscheidungsirrelevante Arbeitszeit (Fahrtzeiten, Aufsuchen und Einmessen der Stichprobenpunkte)


## Zeitstudie:

Mit eingespielten Aufnahmeteams wurde die entscheidungsrelevante Arbeitszeit an einigen Stichprobenpunkten $t_{i}$ ermittelt
Schätzung der fehlenden Daten mittels linearer Regression:

$$
\hat{t}_{i}=\beta_{0}+\beta_{1} \cdot m_{i}
$$

## Arbeitszeiten 2



## Die niedersächsische Betriebsinventur

Mittleres Volumen über alle Straten (Cochran, 1977):

$$
\begin{equation*}
\hat{Y}=\sum_{h=1}^{L} \frac{n_{h}^{\prime}}{n^{\prime}} \hat{Y}_{h}=\sum_{h=1}^{L}\left(\frac{n_{h}^{\prime}}{n^{\prime}} \frac{1}{n_{h}} \sum_{i=1}^{n_{h}} Y_{h i}\right) \tag{12}
\end{equation*}
$$

Varianz (Saborowski et al., 2010):

$$
\begin{equation*}
\widehat{\operatorname{var}}(\hat{Y})=\frac{1}{n^{\prime}-1}\left(\sum_{h=1}^{L} \frac{n_{h}^{\prime}-1}{n^{\prime}} n_{h}^{\prime} \widehat{\operatorname{var}}\left(\hat{Y}_{h}\right)+\sum_{h=1}^{L} \frac{n_{h}^{\prime}}{n^{\prime}}\left(\hat{Y}_{h}-\hat{Y}\right)^{2}\right) \tag{13}
\end{equation*}
$$

## Optimierung von Stichprobenumfang und Allokation

Notwendiger Stichprobenumfang, um eine geforderte Genauigkeit zu erreichen (Cochran, 1977)

- Fixe Arbeitszeit $T$

$$
n=\left(T \sum_{h=1}^{L} n_{h}^{\prime} S_{h} / \sqrt{\bar{t}_{h}}\right)\left(\sum_{h=1}^{L} n_{h}^{\prime} S_{h} \sqrt{\bar{t}_{h}}\right)^{-1}
$$

- Fixer SE

$$
n=\left(\sum_{h=1}^{L} \frac{n_{h}^{\prime}}{n^{\prime}} S_{h} \sqrt{\overline{t_{h}}}\right)\left(\sum_{h=1}^{L} \frac{n_{h}^{\prime}}{n^{\prime}} S_{h} / \sqrt{\bar{t}_{h}}\right)\left(\operatorname{var}(\hat{\bar{Y}})+\frac{1}{n^{\prime}} \sum_{h=1}^{L} \frac{n_{h}^{\prime}}{n^{\prime}} S_{h}^{2}\right)^{-1}
$$

Optimale Allokation der Stichprobenpunkte (Cochran, 1977)

$$
n_{h}=n\left(n_{h}^{\prime} S_{h} / \sqrt{t_{h}}\right)\left(\sum_{h=1}^{L} n_{h}^{\prime} S_{h} / \sqrt{t_{h}}\right)^{-1}
$$

## Zusammenhang von Stichprobenumfang und Schätzgenauigkeit

## Innerhalb eines Stratums gilt:

$$
n_{\text {notwendig }}=\frac{\mathrm{SE}_{\text {Pilotinventur }}^{2} \cdot n_{\text {Pilotinventur }}}{\mathrm{SE}_{\text {gefordert }}^{2}}
$$

bzw.

$$
\mathrm{SE}_{\text {resultierend }}^{2}=\frac{\mathrm{SE}_{\text {Pilotinventur }}^{2} \cdot n_{\text {Pilotinventur }}}{n_{\text {bezahlbar }}}
$$

## Überblick

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## Datensatz

- Zwei simulierte Punktmuster
- Vollständig zufällige räumliche Verteilung (Poisson-Prozess)
- Geklumpte Population (Log-Gauss-Cox-Prozess), angepasst an den "'Hainich Datensatz"' (Bauhus \& Wirth, unveröffentlicht)
- Durchmesserverteilung aus dem Hainich Datensatz abgeleitet (2-parametrige Weibull-Verteilung)
- Entdeckungsfunktion aus den Feldaufnahmen
- 999 Simulationsläufe mit 225 Stichprobenpunkten
- zufällig verteilt
- systematisches Stichprobengitter mit zufälligem Startpunkt


## Punktprozesse

## Poisson-Prozess $N$ :

$N$ zeichnet sich durch zwei Eigenschaften aus (Illian et al., 2008):

- Die Anzahl der Punkte von $N$ in allen finiten Teilmengen $B$ folgt einer Poisson-Verteilung mit Mittelwert $\lambda \nu(B)$.
- Die Anzahl der Punkte von $N$ in $k$ disjunkten Teilmengen bildet $k$ stochastisch unabhängige Zufallsvariablen


## Log-Gauss-Cox-Process (LGCP):

Ein LGCP ist ein inhomogener Poisson-Prozess mit zufälligem Intensitäts-Prozess

$$
\Lambda(x)=\exp (Z(x))
$$

wobei $Z(x)$ ein stationäres und isotropes Gauss-Zufallsfeld ist (Illian et al., 2008). Die Intensität des LGCP ist (Møller \& Waagepetersen, 2003)

$$
\lambda=E \Lambda(x)=\exp \left(\mu+\frac{\theta^{2}}{2}\right)
$$

## Realisation der Punktprozesse

!


