Modeling pulp fiber suspension rheology

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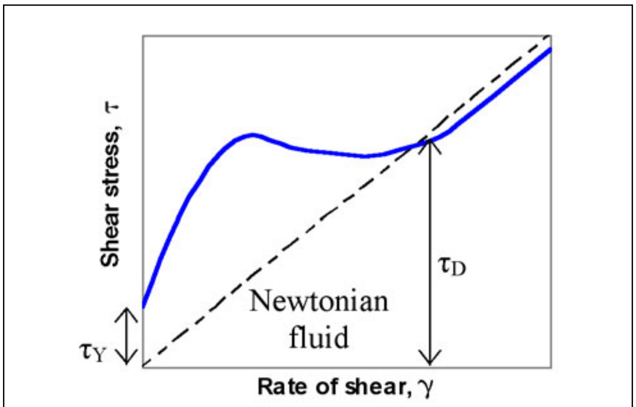
ABSTRACT: Rheological properties of pulp suspensions play a very important role in the industry, mainly due to the consumption of energy for transporting pulp among the different parts of the paper mill. In this work, we determined the rheology of long- and short-fiber bleached kraft pulp suspensions by using a new rotational viscometer especially designed for their analysis. The experimental rheograms were adjusted to the Herschel-Bulkley model. We established the dependence of the rheological parameters on temperature, fiber length, and pulp consistency, to evaluate the relative influence of these factors and the corresponding interactions in the rheological parameters. In addition, we defined a mathematical model that enables the estimation of rheological parameters as a function of the different input factors. We validated this model with some preliminary experimental data.

Application: The knowledge of pulp suspensions rheology is of utmost importance for the optimum design and operation of the majority of unit operations in the pulp and paper industry; accordingly, this study intends to clarify the rheological behavior of pulp fiber suspensions, by modeling it as a function of consistency, temperature and fiber length.

For the optimum design and operation of the majority of unit operations in the mill, knowledge of pulp suspensions rheology is of great significance. For equipment and instrumentation design, it is generally necessary to use some correlations based on the rheological behavior of pulp suspensions.

However, pulp fiber suspensions are different from all other solid-liquid systems, due to the complex interactions between the different components, and the understanding of their rheology remains poor and incomplete.

Figure 1 presents a typical rheogram of pulp fiber suspensions [1].



1. Stress-rate curve for a fiber suspension (adapted from [Gullichsen, 1981]).

Fiber suspensions are coherent networks that possess measurable strength, and external forces must be applied to overcome the network forces. The suspensions begin to move when yield stress (τ_y) has been surpassed. For pulp fiber suspensions, yield stress has typically been correlated with the suspension's consistency (C') [2], as in:

$$\tau_Y = a \times C'^{\,b} \tag{1}$$

where *a* and *b* are empirical parameters that depend on the test conditions and pulp characteristics. Values of *a* between 1.18 and 24.5 and values of *b* between 1.25 and 3.02 have been reported in the literature [2].

If the shear field imposed through the mass of fibers exceeds a shear stress value τ_D, the network structure can be totally disrupted and the suspension will then exhibit a fluid-like behavior. This fluidization point (onset of turbulence) indicates the point where the suspension starts to move in a fully developed turbulent flow and its hydrodynamic properties are similar to those of water.

Duffy and Thitchener [3] expressed the shear stress needed to complete the plug disruption as follows:

$$\tau_D = K' \times C'^{\alpha} \tag{2}$$

For a bleached pine kraft pulp, K′ and α are equal to 2 and 27, respectively. As for the velocity at the onset of turbulence, W_D, it is generally given [4] by:

$$W_D = 1.22 \times C^{1.4} \quad (3)$$

Although pulp suspensions have a clearly non-Newtonian rheological behavior, there is no general agreement concerning the model that should be used. The Herschel-Bulkley model is referred to as one of the most efficient and is widely used for the flow of fiber suspensions [5]. It can be described by the following equations, as in:

$$\tau = \tau_Y + k \times \left(\dot{\gamma}\right)^n \quad , \tau \geq \tau_0$$
$$\dot{\gamma} = 0 \quad , \tau < \tau_0 \quad (4)$$

where τ is the shear stress, τ_Y The yield stress, τ_0 the stress limit above which the fluid will deform, k the consistency coefficient, n the flow behavior index, and $\dot{\gamma}$ the rate of shear.

Our study was designed to obtain additional knowledge about the dynamic behavior of pulp suspensions, and to propose a mathematical model that considers consistency and the effect of temperature and fiber length on the rheological parameters.

MATERIALS AND METHODS

An experimental design was established to evaluate the influence of temperature, consistency, and fiber length on the rheological parameters of short- and long-fiber pulp suspensions.

We tested two types of fibers: short eucalyptus fibers and long pine fibers. We used a three-level factorial design (**Table I**) and a design with 27 experiments (3^3) for both. The different fiber lengths were obtained by beating the unbeaten pulps in the laboratory, at different intensities.

To evaluate the rheology of the pulp suspensions, we used a new plate rotational viscometer, based on the Searle effect and especially built up to maintain a uniform fiber distribution [6,7]. This viscometer measures the shear in the rotor (mobile plates) and in the vessel (fixed plates), and thus the torque applied by the rotor and the torque transmitted by the fluid to the vessel can be quantified.

Factor		Level (-1)	Level (0)	Level (1)
Consis - A		1%	2%	3%
Fiber Length - B	Short fiber	500 µm S (-1)	580 µm S (0)	660 µm S (1)
	Long fiber	1190 µm L (-1)	1580 µm L (0)	1745 µm L (1)
Temperature - C		283 K	296 K	310 K

I. Experimental design for the short- and long-fiber pulp suspensions. (Consis = consistency.)

The Herschel-Bulkley model was adopted to analyze the rheograms and to define a rheological model describing the experimental data points. In this study, the equation (4) was linearized to obtain τ_Y, K , and n.

For τ_Y, we considered the first data point of the experimental rheogram (τ_Y [5 rad/s]) and the yield stress value extrapolated from the rheogram to a zero shear rate (τ_Y [extrap]). Additionally, the disruptive velocity, W_D, and the disruptive shear stress, τ_D, were acquired from all rheograms at the onset of turbulence point.

The results were analyzed using the Design Expert© software. The relative importance of the different factors and possible interactions between them were determined. Further, for each rheological parameter, a design equation that enables its estimation as a function of consistency, fiber length, and temperature was established.

RESULTS

The results of the statistical analysis of τ_Y (5 rad/s), for the short fiber suspensions, are summarized in **Table II**.

Considering the coded factors, the significant effects and interactions, in order of decreasing relevance, are consistency, consistency^2, consistency/temperature, temperature, fiber length, and consistency/fiber length. Some factors have a positive effect (consistency, consistency^2, fiber length, and consistency/temperature) and others have a negative effect (consistency/temperature and temperature) in the response.

In terms of the actual factors, we observed that the signals of some factors change when compared with the predicting equation's coded factors. However, the global effect of individual factors remains exactly the same as in the equation with the coded factors, because the combined factors balance the overall response. This is due to the different scale factors (consistency, fiber length, and temperature) used

Equation in terms of the coded factors - relative factor importance
$\tau_Y = 22.16 + 25.52 \cdot A + 3.689 \cdot B - 4.345 \cdot C + 1.855 \cdot AB - 4.639 \cdot AC - 0.407 \cdot BC + 9.037 \cdot A^2 - 0.942 \cdot B^2 - 1.399 \cdot C^2$ $A, B, C, A \cdot B, A \cdot C, A^2$ are significant factors for the model
Equation in terms of the actual factors - response surface
$\tau_Y = -897 + 77.3 \cdot Cons + 3.02x10^{1} \cdot L_{fiber} + 5.15 \cdot T + 2.40x10^{3} \cdot Cons \cdot L_{fiber} - 3.44x10^{3} \cdot Cons \cdot T - 3.90x10^{-2} \cdot L_{fiber} \cdot T + 9.04 \cdot Cons^2 + 1.59x10^{4} \cdot L_{fiber}^2 + 7.68x10^{3} \cdot T^2$ $R^2 = 0.988$ $Cons \in [1\%, 3\%]; \quad L_{fiber} \in [500\mu m, 660\mu m]; \quad T \in [283K, 310K]$

II. Design equations for τ_Y(5 rad/s) for the short fiber pulp suspensions.

Factors	Rheological Parameters					
	$\tau_Y(5 rad/s)$	$\tau_Y(extrap)$	τ_D	W_D	n	k
Constant	22.2	21.9	37.4	207	0.75	0.11
A	25.5	22.8	67.0	111	-0.13	0.12
B	3.69	2.05	10.7	24	-0.03	0.04
C	-4.34	-4.37	-4.39	0	0.02	-0.04
AB	1.86	-0.02	8.14	0	-	-
AC	-4.64	-5.08	-5.61	0	-	-
BC	-0.41	-0.28	0.96	0	-	-
A^2	9.04	7.59	-	-	-	-
B^2	-0.94	-5.19	-	-	-	-
C^2	-1.40	-1.14	-	-	-	-

III. Coefficients of the coded factors for each parameter for the short-fiber pulp suspensions.

Factors	Rheological Parameters					
	$\tau_Y(5 rad/s)$	$\tau_Y(extrap)$	τ_D	W_D	n	k
Constant	-8.97E+02	-1.03E+03	-3.62E+02	-1.96E+02	8.94E-01	3.54E-01
Consis (%)	7.73E+01	1.04E+02	1.29E+02	1.11E+02	-1.26E-01	1.15E-01
L_{fiber} (μm)	3.00E-01	1.12	2.00E-01	3.12E-01	-4.26E-04	5.26E-04
T (K)	5.15	4.31	1.04	0	1.20E-03	-2.63E-03
Consis·L_{fiber}	2.40E-03	-2.16E-04	1.06E-01	0	-	-
Consis·T	-3.44E-01	-3.77E-01	-4.15E-01	0	-	-
L_{fiber}·T	-3.90E-04	-2.73E-04	-9.19E-04	0	-	-
$Consis^2$	9.04	7.59	-	-	-	-
L_{fiber}^2	1.59E-04	-8.70E-04	-	-	-	-
T^2	7.68E-03	-6.28E-03	-	-	-	-
R^2	0.988	0.988	0.992	0.964	0.797	0.564

IV. Coefficients of the actual factors for each parameter for the short-fiber pulp suspensions. (Consis = consistency.)

Factors	Rheological Parameters					
	$\tau_Y(5 rad/s)$	$\tau_Y(extrap)$	τ_D	W_D	n	k
Constant	57.7	48.4	73.33	218	0.76	0.14
A	43.4	36.4	108	79.8	-0.07	0.11
B	5.74	6.23	21.67	32.0	6.22E-04	0.03
C	-3.72	-3.26	-6.00	-8.00	-9.98E-03	-0.02
AB	1.31	2.85	3.33	15.5	-	-
AC	-1.88	-5.13	-4.50	15.3	-	-
BC	-0.95	-1.18	-0.25	6.00	-	-
A^2	9.01	5.08	-	-	-	-
B^2	-12.2	-8.39	-	-	-	-
C^2	-2.20	-1.24	-	-	-	-

V. Coefficients of the coded factors for each parameter for the long-fiber pulp suspensions.

factors (consistency, fiber length, and temperature) used when changing from coded to actual factors.

A similar analysis can be done for all the other parameters (τ_Y[extrap], τ_D, W_D, k, and n). The corresponding results are presented in **Tables III** and **IV** for the short-fiber pulp suspensions and in **Tables V** and **VI** for the long-fiber pulp suspensions.

In addition, we considered the unbeaten short- and long-fiber suspensions all together in the statistical analysis. The corresponding results are listed in **Table VII**.

From the analysis of Tables II to VII, some major conclusions can be drawn:

- The behavior of τ_Y (τ_Y[5 rad/s] and τ_Y[extrap]) is analogous for both types of fibers. From the comparison of the coefficients of the coded factors (Tables II and V), we concluded that the impacts of each factor in the response remain very similar. Additionally, a careful comparison with Table VII suggests that this parameter seems to show an almost continuous trend along fiber length, from the shorter fiber length (S[-1]) to the longer fiber length (L[1]).
- τ_D also shows equivalent behavior for the short- and long-fiber pulp suspensions: τ_D increases with consistency and fiber length. In fact, these are the most important factors in the response, as confirmed by the coefficients of the coded factors (Tables III and V). We may also conclude, by comparison with Table VII, that this parameter presents almost continuous behavior along the fiber length scale in this case.
- The velocity of the onset of turbulence, W_D, increases with fiber length both for the short- and the long-fiber pulp suspensions (Tables III and V). However, when considering these pulp suspensions together in the statistical analysis (Table VII), the influence of fiber length on W_D is not detected. Therefore, the variation of this parameter is not independent of the

are different whether short or long fibers are considered. (The behavior cannot be extrapolated on the length scale independently of the fiber's characteristics tested.)

- The variation of the parameters n and k is also similar for both kind of fibers tested. n is mainly influenced by consistency, and a careful analysis of the equations shows that the influence of this factor is stronger for the short fibers than for the long fibers. Therefore, as consistency increases, the rheology of the short fiber pulp suspensions gets further away from the Newtonian behavior, when compared with the long fibers.
- For every rheological parameter, consistency is most important.
- As a result of the increase of the interactions between fibers for higher values of consistency, a greater stress is necessary to break the fibrous network, and therefore yield stress increases. Also, more energy is necessary to fluidize the suspensions, and thus velocity and shear stress for the onset of turbulence increase.
- The difference between the rheological parameters of the pulp suspensions and the Newtonian parameters increases with consistency because the fiber suspensions present complex shear mechanisms due to the interactions between the fiber network and the different components of these suspensions.
- Temperature has a negative effect on yield stress and on disruptive shear stress. In fact, as temperature increases, less energy is required to break up the fiber networks (so that the suspensions start to move) and to overcome friction between different fiber flocs (so that suspensions start to flow like a fluid).
- The increase of fiber length pro-

Factors	Rheological Parameters					
	$\tau_y(5rad/s)$	$\tau_y(extrap)$	τ_D	W_D	n	k
Constant	-1.52E+03	-9.93E+02	-3.16E+02	1.59E+03	1.10	2.55E-01
Consis (%)	4.17E+01	4.78E+01	1.89E+02	-3.38E+02	-6.99E-02	1.11E-01
L_{fiber} (µm)	5.44E-01	4.11E-01	7.30E-02	-4.69E-01	2.23E-06	1.20E-04
T (K)	7.53	4.55	3.20E-01	-5.20	-7.39E-03	-1.74E-03
Consis·L_{fiber}	4.70E-03	1.02E-02	1.20E-02	5.60E-02	-	-
Consis·T	-1.39E-01	-1.58E-01	-3.33E-01	1.10	-	-
L_{fiber}·T	-2.50E-04	-3.10E-04	-6.64E-05	1.59E-03	-	-
Consis^2	9.01	5.08	-	-	-	-
L_{fiber}^2	-1.56E-04	-1.08E-04	-	-	-	-
T^2	-1.21E-02	-6.80E-03	-	-	-	-
R^2	0.991	0.995	0.998	0.889	0.750	0.793

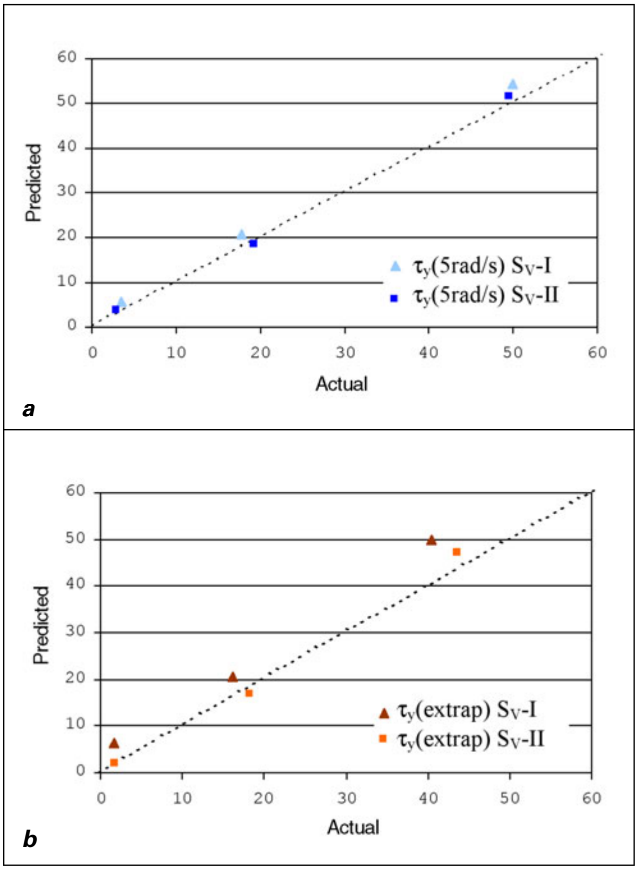
VI. Coefficients of the actual factors for each parameter and for the long-fiber pulp suspensions. (Consis = consistency.)

Factors	Rheological Parameters					
	$\tau_y(5rad/s)$	$\tau_y(extrap)$	τ_D	W_D	n	k
Constant	38.4	35.3	71.0	239	0.75	0.18
A	35.4	29.4	92.8	95.0	-0.09	0.19
B	12.3	12.9	240.0	0	0.02	-0.04
C	-4.35	-4.83	-5.50	0	0.01	-0.01
AB	7.76	8.93	18.5	0	-	-
AC	-3.54	-4.88	-5.75	0	-	-
BC	0.04	0.33	-0.13	0	-	-
A^2	7.59	3.11	-	-	-	-
B^2	-	-	-	-	-	-
C^2	-0.85	-2.17	-	-	-	-

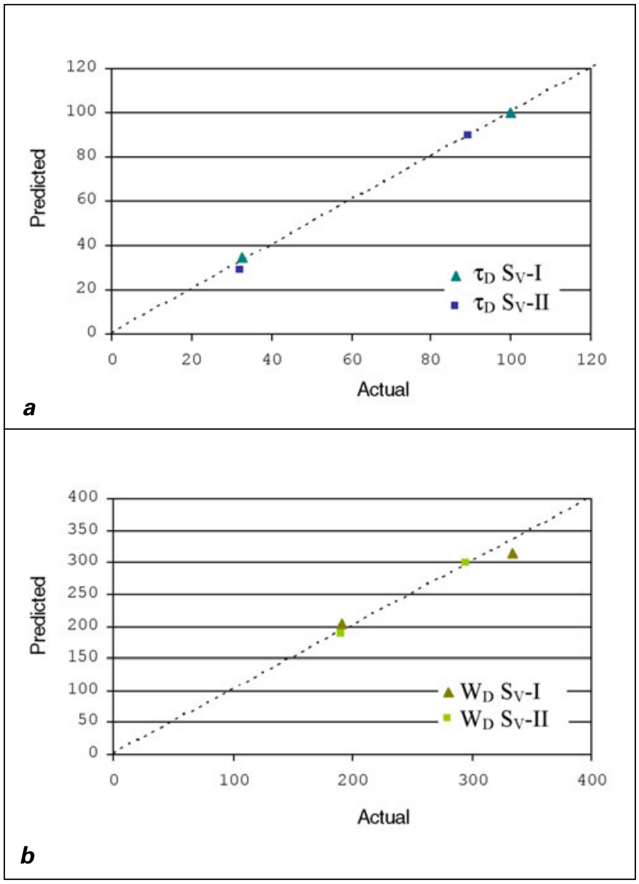
VII. Coefficients of the coded factors for each parameter for the unbeaten short- and long-fiber pulp suspensions.

Short fiber suspension	Characteristics	Test Conditions	
	Mean Fiber length (µm)	Consis (%)	Temperature (K)
S_v-I	570	1, 2 and 3	296
S_v-II	520		

VIII. Characteristics and test conditions for short-fiber pulp suspensions used in the validation of the design equations. (Consis = consistency.)



2. Predicted vs actual values for: a) τ_y(5 rad/s) and b) τ_y(extrap) for two distinct short-fiber suspensions (Sv-I and Sv-II).



3. Predicted vs actual values for a) τ_D and b) W_D for two distinct short-fiber suspensions (Sv-I and Sv-II).

motes higher yield stress and disruptive shear stress, because, as expected, there are more contact points between longer fibers and, consequently, more energy is necessary to break the fiber network.

- The influence of temperature and fiber length on k and *n* is not so clear.
- Finally, we emphasize that the coefficients of correlation (R^2) are good for most of the rheological parameters (Tables IV and VI), the exceptions being for parameters *n* and k.

Models testing

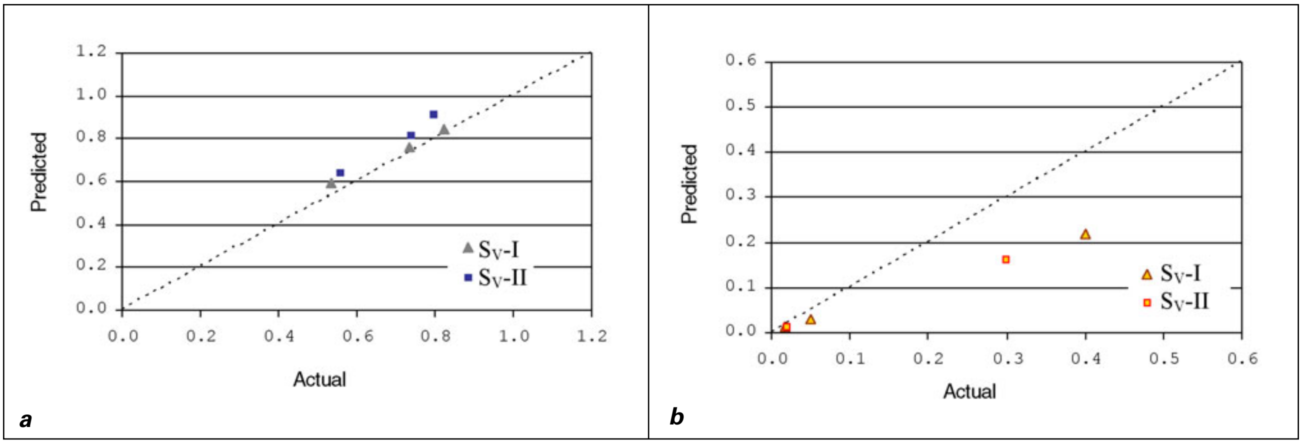
After the statistical analysis of the results, we performed tests to validate the models, using preliminary experimental data obtained with two distinct short fiber suspensions (S_v-I and S_v-II). The characteristics of the suspensions and the test conditions are presented in **Table VIII**, and the results are plotted in **Figs. 2** to **4**.

Figures 2 and 3 show that, in general, the model equations can reproduce the new experimental data without major errors. Therefore, we can conclude that they have a good prediction capacity and can be used to estimate future responses for the same boundary conditions. For the parameters *n* and k (Fig. 4), the model does not predict the values for the new suspensions as well, especially in the case of *k*. However, we must remember that both parameters are estimates and the corresponding design equations have low correlation coefficient values. These facts contribute to the poorer prediction potential.

CONCLUSIONS

The results confirm that the pulp suspensions present non-Newtonian behavior and their rheology is strongly influenced by consistency. In fact, all the models produced reveal that consistency (individually or combined with fiber length or temperature) is the factor that most affects the rheological parameters. This confirms the development of flocs and floc networks of fibers for the higher consistencies. Further, the increase in consistency increases the difference of the Newtonian rheological behavior.

The rheology of the pulp suspensions is also affected by temperature and fiber length. Temperature decreases yield stress and disruptive shear stress of pulp suspensions, while fiber length has an opposite impact on these parameters. The



4. Predicted vs actual values for a) n and b) k for two distinct short fiber suspensions (Sv-I and Sv-II).

influence of temperature and fiber length on the rheological parameters n and k is not so clear.

In general, there is no fundamental difference between the rheology of short- and long-fiber pulp suspensions. However, the influence of consistency in short-fiber suspensions is stronger than in long fiber suspensions.

The close agreement between new experimental and predicted results reveals that the models that have been established can be used with a reasonable degree of accuracy, if the same limits for each factor are assumed. **TJ**

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NOMENCLATURE

a, b - constants

A, B, C, AB, AC, BC, A^2, B^2, C^2 - coded model factors

C', Consist - consistency (%)

K', α - constants

κ - consistency coefficient

L - long fiber

L_{fiber} - fiber length (µm)

n - flow behavior index

S - short fiber

R^2 - coefficient of correlation

T - temperature (K)

W_D - velocity at the onset of turbulence point (m/s)

τ - shear stress (Pa)

τ_D - disruptive shear stress (Pa)

τ_Y - yield stress (Pa)

τ_0 - the stress limit above which the fluid will deform (Pa)

$\dot{\gamma}$ - shear rate (s^{-1})

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INSIGHTS FROM THE AUTHORS

Interest in particle technology and multiphase flows led to the investigation of the relation between particle characteristics and rheological behavior of suspensions. As fibers can be considered as particles, it was just a question of extending investigation into an application important to our industrial partners.

Fibers behave quite differently from other types of particles and, thus, it was necessary to use a rheometer designed to deal with fiber suspension. We found no published information on the dependence of rheological parameters on fiber characteristics and consistency, nor did we find in the literature any similar study that used statistical design to produce the most relevant models.

The most difficult aspect of this work was the experimental part and all its preparation, because it was necessary to perform 27 experiments for each fiber-type experimental design. This involved several weeks of hard work.

From this research we discovered that consistency is the factor that most affects the rheological parameters, but temperature, as expected, and fiber length also play a role in the rheology of fiber suspensions. The most interesting finding lies in the fact that there is no fundamental difference between the rheology of short- and long-fiber suspensions. Some rheological parameters present an almost continuous behavior along the fiber length scale.

Knowledge of pulp suspensions rheology is of utmost importance for optimum design and operation of most unit operations in pulp and paper mills. Readers can use the information produced in this study to predict rheological properties of pulp suspension for more accurate designing of equipment and instrumentation. The next step will be to incorporate the research findings in the modeling of the flow of fibre suspensions.

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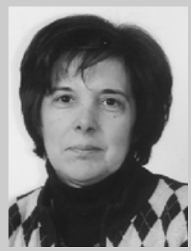

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